Determine the order and degree, if defined, of the following differential equations. State also, if these are linear or non-linear:

1. (*i*) $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$

(*ii*) $\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0$

(*iii*) $\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$

(*iv*) $\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0$

(*v*) $\left(\frac{d^2s}{dt^2}\right)^2 + \left(\frac{ds}{dt}\right)^3 + 4 = 0$

(*vi*) $y\left(\frac{d^4y}{dx^4}\right)^4 = 7$

2. (*i*) $5x\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2} - 6y = \log x$

(*ii*) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = x\left(\frac{d^3y}{dx^3}\right)^2$

(*iii*) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

(*iv*) $y''^2 - 2y'' - y' + 1 = 0$

(*v*) $t^2\frac{d^2s}{dt^2} - st\frac{ds}{dt} = \frac{d^4s}{dt^4}$

(*vi*) $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

3. (*i*) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 5\frac{d^2y}{dx^2}$

(*ii*) $\sqrt{1-x^2}\,dx + \sqrt{1-y^2}\,dy = 0$

(*iii*) $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

(*iv*) $y = px + \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$

(*v*) $x\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$

(*vi*) $\sin\left(\frac{d^2y}{dx^2}\right) + \frac{dy}{dx} = 9$

4. (*i*) $\frac{d^4y}{dx^4} + \sin(y''') = 0$

(*ii*) $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

VERY SHORT ANSWER TYPE QUESTIONS

1. Show that $x^2 + 4y = 0$ is a solution of $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$.

2. Show that $y = \sqrt{1+x^2}$ is a solution of $y' = \frac{xy}{1+x^2}$.

3. Show that $y = \frac{1}{2x} + Ax + B$ is a solution of $x^3\frac{d^2y}{dx^2} = 1$.

4. Show that $y = a\cos x + b\sin x$ is a solution of $y'' + y = 0$.

5. Show that $y = 3\cos(\log x) + 4\sin(\log x)$ is a solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

6. Show that $y = ae^{2x} + be^{-x}$ is a solution of $y_2 - y_1 - 2y = 0$.

7. Show that $y = e^{3x}(A + Bx)$ is a solution of $y_2 - 6y_1 + 9y = 0$.

8. Show that $y = c_1e^{ax}\cos bx + c_2e^{ax}\sin bx$ is a solution of $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

9. Show that $y = \cos(\cos x)$ is a solution of $\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} + y\sin^2 x = 0$.

10. Show that $x + y = \tan^{-1} y$ is a solution of $y^2y' + y^2 + 1 = 0$.

11. Show that $y = x\sin x$ is a solution of $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0$ and $x > y$ or $x < -y)$.

12. Show that $y - \cos y = x$ is a solution of $(y\sin y + \cos y + x)y' = y$.

13. Show that $x^2 = 2y^2\log y$ is a solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$.

14. Show that $y = c_1e^x + c_2e^{-x}$ is the general solution of $\frac{d^2y}{dx^2} - y = 0$.

15. Show that $y = e^x + 1$ is a solution of $y'' - y' = 0$.

16. Show that $y = x^2 + 2x + C$ is a solution of $y' - 2x - 2 = 0$.

17. Show that $y = \cos x + C$ is a solution of $y' + \sin x = 0$.

18. Show that $y = Ax$ is a solution of $xy' = y$, $x \neq 0$.

19. Show that $y = ae^x + be^{-x} + x^2$ is a solution of $\frac{d^2y}{dx^2} - y + x^2 - 2 = 0$.

LONG ANSWER-I TYPE QUESTIONS

Solve the following differential equations (Q. No. 1–25):

1. $(3xy + y^2)dx + (x^2 + xy)dy = 0$

2. $2xyy' = x^2 + 3y^2$

3. $(x^2 + xy)dy = (x^2 + y^2)dx$

4. $(x^2 - y^2)dx + 2xy\,dy = 0$

5. $x\dfrac{dy}{dx} + \dfrac{y^2}{x} = y$

6. $x^2y\,dx - (x^3 + y^3)dy = 0$

7. $x^2\dfrac{dy}{dx} = y(x + y)$

8. $y - x\dfrac{dy}{dx} = x + y\dfrac{dy}{dx}$

9. $x^2\dfrac{dy}{dx} = \dfrac{y(x + y)}{2}$

10. $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$

11. $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$

12. $(x - y)\dfrac{dy}{dx} = x + 2y$

13. $x^2y_1 = x^2 - 2y^2 + xy$

14. $(x^2 - y^2)dx + xy\,dy = 0$

15. $x^2\,dy + y(x + y)dx = 0$

16. $y\,dx + x\left(\log\dfrac{y}{x}\right)dy - 2x\,dy = 0$

17. $xy\left(\log\dfrac{y}{x}\right)dx + \left(y^2 - x^2\log\dfrac{y}{x}\right)dy = 0$

18. $\dfrac{y}{x}\cos\dfrac{y}{x}\,dx - \left(\dfrac{x}{y}\sin\dfrac{y}{x} + \cos\dfrac{y}{x}\right)dy = 0$

19. $(y^2 - 2xy)dx = (x^2 - 2xy)dy$

20. $y^2\,dx + (x^2 - xy + y^2)dy = 0$

21. $2xy\,dx + (x^2 + 2y^2)\,dy = 0$

22. $(y^2 - x^2)dy = 3xy\,dx$

23. $x\cos\left(\dfrac{y}{x}\right)\dfrac{dy}{dx} = y\cos\left(\dfrac{y}{x}\right) + x$

24. $(x - y)dy - (x + y)dx = 0$

25. $x\dfrac{dy}{dx} - y + x\sin\dfrac{y}{x} = 0.$