Long Answer-I Type Questions (4 Marks)

1. Express $\frac{2-\sqrt{-25}}{1-\sqrt{-16}}$ in the form $a + ib$. Also, find its conjugate.
2. Find the value of x and y if $(x+iy)(2-3i)=4+i$
3. If $(x+iy)^{\frac{1}{3}}=a+ib$, $x, y, a, b \in R$, show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$
4. If $a+ib=\frac{c+i}{c-i}$, where c is real, prove that $a^2+b^2=1$ and $\frac{b}{a}=\frac{2c}{c^2-1}$
5. If $\frac{(a+i)^2}{(2a-i)}=p+iq$, show that $p^2+q^2=\frac{(a^2+1)^2}{4a^2+1}$
6. Find the square roots of $-48 - 14i$
7. Find the least value of positive integer m so that $\left(\frac{1+i}{1-i}\right)^m=1$
8. If $x=-5+2\sqrt{-4}$, find the value of $x^4+9x^3+35x^2-x+4$
9. Express $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$ in the form $a + ib$.
10. Find the multiplicative inverse of $\frac{(i+1)(i+2)}{(i-1)(i-2)}$
11. Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$
12. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$
13. Find the number of non-zero integral solutions of $|1-i|^x = 2^x$
14. Find the values of x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.
15. Solve the equation $27x^2 - 10x + 1 = 0$