1) Show that:

(i) $\tan^{-1}\frac{2}{3} = \frac{1}{2}\tan^{-1}\frac{12}{5}$

(ii) $2\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{4}{3}$

(iii) $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

(iv) $\tan^{-1}\frac{3a^2x - x^3}{a^3 - 3ax^2} = 3\tan^{-1}\frac{x}{a}, \quad -\frac{a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}, a > 0.$

2) Show that:

(i) $2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

(ii) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \sin^{-1}\frac{31}{25\sqrt{2}}$

(iii) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3} = \frac{1}{2}\cos^{-1}\frac{3}{5} = \frac{1}{2}\sin^{-1}\frac{4}{5}.$

3) Show that:

(i) $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

(ii) $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

(iii) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$

(iv) $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}.$

4) Solve the equations:

(i) $\tan^{-1}\sqrt{3} + \cot^{-1} x = \pi/2$

(ii) $\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}$, $0 < x < \sqrt{6}$

(iii) $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, $0 < x < \frac{1}{\sqrt{6}}$

(iv) $\tan^{-1}(x + 2) + \tan^{-1}(x - 2) = \tan^{-1}\frac{8}{79}$, $x > 0$.

5) Show that:

(i) $\tan^{-1} t + \tan^{-1} \dfrac{2t}{1-t^2} = \tan^{-1} \dfrac{3t - t^3}{1-3t^2}, \; t^2 < \dfrac{1}{3}$

(ii) $\tan^{-1} \dfrac{m}{n} - \tan^{-1} \dfrac{m-n}{m+n} = \dfrac{\pi}{4}$

(iii) $\tan^{-1} n + \cot^{-1} (n+1) = \tan^{-1} (n^2 + n + 1)$

(iv) $\tan^{-1} \dfrac{1-x}{1+x} - \tan^{-1} \dfrac{1-y}{1+y} = \sin^{-1} \dfrac{y-x}{\sqrt{1+x^2}\,\sqrt{1+y^2}}$.

6) Show that: $\tan^{-1}\left(\dfrac{1}{2}\tan 2A\right) + \tan^{-1} (\cot A) + \tan^{-1} (\cot^3 A) = 0, \; \dfrac{\pi}{4} < A < \dfrac{\pi}{2}$.

7) Show that: $\tan^{-1} \dfrac{2ab}{a^2 - b^2} + \tan^{-1} \dfrac{2xy}{x^2 - y^2} = \tan^{-1} \dfrac{2\alpha\beta}{\alpha^2 - \beta^2}$,

where $\alpha = ax - by$ and $\beta = ay + bx$.

8) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

9) Show that:

(i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

(ii) $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} = \tan^{-1}\frac{2}{9}$

(iii) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

(iv) $\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$

(v) $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

(vi) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

(vii) $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3.$