1) $a + (a + d) + (a + 2d) + \cdots\cdots + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d], n \in \mathbf{N}.$

2) $\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \cdots\cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}, n \in \mathbf{N}.$

3) $\frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots\cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}, n \in \mathbf{N}.$

4) $\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots\cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}, n \in \mathbf{N}.$

5) $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \cdots\cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}, n \in \mathbf{N}.$

6) $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}, n \in \mathbf{N}.$

7) $1\cdot 2 + 2\cdot 2^2 + 3\cdot 2^3 + \cdots\cdots + n\cdot 2^n = (n-1)2^{n+1} + 2, n \in \mathbf{N}.$

8) $1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + 3\cdot 4\cdot 5 + \cdots\cdots + n(n+1)(n+2)$

$$= \frac{n(n+1)(n+2)(n+3)}{4}, n \in \mathbf{N}.$$

9) $1 + 2 + 2^2 + \cdots\cdots + 2^n = 2^{n+1} - 1, n \in \mathbf{N}.$

10) $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}, n \in \mathbf{N}.$

11) $1 + x + x^2 + \cdots\cdots + x^n = \frac{1 - x^{n+1}}{1 - x}, n \in \mathbf{N}.$

12) $a + ar + ar^2 + \cdots\cdots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, n \in \mathbf{N}.$

13) $1 + 3 + 3^2 + \cdots\cdots + 3^{n-1} = \frac{3^n - 1}{2}, n \in \mathbf{N}.$

14) $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\cdots\left(1+\frac{1}{n}\right) = n + 1, n \in \mathbf{N}.$

15) $1^2 + 3^2 + 5^2 + \cdots\cdots + (2n-1)^2 = \frac{n(4n^2 - 1)}{3}, n \in \mathbf{N}.$

16) $1^3 + 2^3 + 3^3 + \cdots\cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2, n \in \mathbf{N}.$

17) $\sin\theta + \sin 3\theta + \cdots\cdots + \sin(2n-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}, n \in \mathbf{N}.$

18) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}, n \geq 2$ and $n \in \mathbf{N}.$

19) $1(1!) + 2(2!) + 3(3!) + \cdots\cdots + n(n!) = (n+1)! - 1, n \in \mathbf{N}.$

20) A sequence $b_0, b_1, b_2, \cdots$ is defined as $b_0 = 5$ and $b_k = 4 + b_{k-1}, k \in \mathbf{N}$. Show that $b_n = 5 + 4n, n \in \mathbf{N}$ using mathematical induction.

21) A sequence $a_1, a_2, a_3, \cdots$ is defined as $a_1 = 3$ and $a_k = 7a_{k-1}$ for all natural numbers $k \geq 2$. Show that $a_n = 3 \cdot 7^{n-1}, n \in \mathbf{N}.$